Code No: **R231105** **R23** **SET-1**

I B. Tech I Semester Regular Examinations, January-2024
LINEAR ALGEBRA AND CALCULUS
(Common to all Branches)

Time: 3 hours Max. Marks: 70

*Note: 1. Question paper consists of two parts (**Part-A** and **Part-B**)*
*2. All the questions in **Part-A** is Compulsory*
*3. Answer **ONE** Question from **Each Unit** in **Part-B***

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PART –A (20 Marks)

1. a) Define linear system of equations. [2M]
   b) What is the normal form? [2M]
   c) Find the matrix corresponding to quadratic form $x^2 + 4xy + 2y^2$. [2M]
   d) Find the sum of the Eigen values of matrix $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ [2M]
   e) State Cauchy's mean value theorem. [2M]
   f) Write the geometrical interpretation for Lagrange's mean value theorem. [2M]
   g) Find $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ for $f(x, y) = xy + x^2 + 2y$ [2M]
   h) Find $\frac{\partial u}{\partial x}$ if $u = f(x + y, x - y)$ [2M]
   i) Let $f(x, y)$ be a continuous function in R^2 where $R = \{(x, y)/\ a \le x \le b;\ c \le y \le d\}$ then $\iint_R f(x, y)\ dx\ dy = ?$ [2M]
   j) Evaluate $\int_0^1 \int_0^2 xy\ \ dx\ dy$ [2M]

PART – B (50 MARKS)

UNIT-I

2. a) Find the rank of the matrix using echelon form $\begin{bmatrix} 1 & 2 & 3 & -2 \\ 2 & -2 & 1 & 3 \\ 3 & 0 & 4 & 1 \end{bmatrix}$ [5M]
   b) Solve the system of equations using Gauss elimination method [5M]
   $10x + y + z = 12,\ 2x + 10y + z = 13,\ x + y + 5z = 7.$

(OR)

3. a) Find the inverse using Gauss-Jordan method $\begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$ [5M]
   b) Find the values of 'a' and 'b' for which the system of equations [5M]
   $x + y + z = 3, x + 2y + 2z = 6, x + ay + 3z = b$ has a unique solution.

UNIT-II

4. a) Determine the eigen values of $adjA$ where $A = \begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$ [5M]
   b) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ [5M]
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(OR)

5. Diagonalize the matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ and find A^4 using model matrix 'P' [10M]

UNIT-III

6. a) Verify Rolle's mean value theorem $f(x) = \frac{sinx}{e^x}$ in $[0, \pi]$ [5M]

b) Write the Taylor's series expansion for $f(x) = \log(1 + x)$ about $x = 0$ [5M]

(OR)

7. Prove that $\frac{\pi}{6} + \frac{1}{5\sqrt{3}} < sin^{-1}\left(\frac{3}{5}\right) < \frac{\pi}{6} + \frac{1}{8}$ [10M]

UNIT-IV

8. a) If $x = r\cos\theta, y = r\sin\theta$ then prove that $\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} = 0$ [5M]

b) Determine whether the following functions are functionally dependent if so find the relation between $if\ u = x\sqrt{1 - y^2} + y\sqrt{1 - x^2},\ v = sin^{-1}(x) + sin^{-1}(y)$. [5M]

(OR)

9. a) Find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ if $u = \log\left(\frac{x^2+y^2}{xy}\right)$ [5M]

b) Find extreme values$f(x, y) = 1 - x^2 - y^2$ [5M]

UNIT-V

10. By change of Integration Evaluate [10M]

$$\int_o^1 \int_{x^2}^x xy\ dx\ dy$$

(OR)

11. Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$ using triple integration. [10M]

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Code No: **R231105** **R23** **SET-2**

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Time: 3 hours Max. Marks: 70

*Note: 1. Question paper consists of two parts (**Part-A** and **Part-B**)*
*2. All the questions in **Part-A** is Compulsory*
*3. Answer **ONE** Question from **Each Unit** in **Part-B***

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PART –A (20 Marks)

1. a) Define the rank of the matrix [2M]
   b) If the matrix of order $(m \times n)$, then that would be the rank of the matrix [2M]
   c) Find the product of the Eigen values of $\begin{bmatrix} 2 & 2 \\ 2 & 6 \end{bmatrix}$ [2M]
   d) Find the Eigen vector corresponding to $\lambda = 5$ for the matrix $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$ [2M]
   e) Write the geometrical interpretation for Rolle's mean value theorem. [2M]
   f) Find the value of 'c' using Lagrange's mean value theorem for $f(x) = x^2\ in\ [1,5]$ [2M]
   g) Find $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ for $f(x,y) = e^{xy} + 2x^2$ [2M]
   h) Find $\frac{\partial u}{\partial y}$ if $u = f(x+y, x-y)$ [2M]
   i) If $f(x, y)$ be a continuous function defined aver region R, where $R = \{(x, y)/\ a \le x \le b$ and $y_1 \le y \le y_2\}$ then $\iint_R f(x,y)\ dx\ dy = ?$ [2M]
   j) Evaluate $\int_0^1 \int_0^1 e^{xy}\ dx\ dy$ [2M]

PART – B (50 MARKS)

UNIT-I

2. a) Solve the system of equations using Gauss elimination method $3x + y + 2z = 3, 2x - 3y - z = -3, x + 2y + z = 4$ [5M]
   b) Find the inverse using Gauss-Jordan method $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ [5M]

(OR)

3 a) Find the rank of the matrix using Normal form $\begin{bmatrix} 1 & 3 & 6 & -1 \\ 1 & 4 & 5 & 1 \\ 1 & 5 & 4 & 3 \end{bmatrix}$ [5M]
   b) Test the consistency, if so, solve the system of equations $x + y + z = 6, x + 2y + 3z = 10, x + 2y + 3z = 5$ [5M]

UNIT-II

4. a) Determine the Eigen values of A^{-1} where $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ [5M]
   b) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 2 & 4 & 7 \\ 0 & 1 & 8 \\ 0 & 0 & 3 \end{bmatrix}$ [5M]
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(OR)

5. Diagonalize the matrix $A = \begin{bmatrix} 2 & 2 & -7 \\ 2 & 1 & 2 \\ 0 & 1 & -3 \end{bmatrix}$ and find A^4 using model matrix 'P' [10M]

UNIT-III

6. a) Write the Taylor's series expansion for $f(x) = \log(1-x)$ about $x = 0$ [5M]

b) Verify Rolle's mean value theorem $f(x) = |x|$ in $[-1,1]$ [5M]

(OR)

7. If $a < b$ prove that $\frac{b-a}{1+b^2} < tan^{-1}b - tan^{-1}a < \frac{b-a}{1+a^2}$ [10M]

UNIT-IV

8. a) Expand $f(x,y) = xy^2 + \cos(xy)$ in powers of $(x-1)$ and $(y-\frac{\pi}{2})$ using Taylor's series. [5M]

b) If $u = \frac{y}{z} + \frac{z}{x}$ then find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$ [5M]

(OR)

9. a) Find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ if $u = sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$ [5M]

b) Show that $u = \frac{x}{y-z}, v = \frac{y}{z-x}, w = \frac{z}{x-y}$ are functionally dependent. [5M]

UNIT-V

10. Evaluate by change of order of Integration [10M]

$$\int_0^{2a} \int_{y^2/4a}^{3a-y} dx\ dy$$

(OR)

11. Evaluate $\iiint_R z(x^2 + y^2)\ dx\ dy\ dz$ where R is the Region bounded by the cylinder. $x^2 + y^2 = 1$ and the planes $z = 2$ and $z = 3$ by changing it to cylindrical coordinates. [10M]

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Code No: **R231105** **R23** **SET-3**

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Time: 3 hours Max. Marks: 70

*Note: 1. Question paper consists of two parts (**Part-A** and **Part-B**)*
*2. All the questions in **Part-A** is Compulsory*
*3. Answer **ONE** Question from **Each Unit** in **Part-B***

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PART –A (20 Marks)

1. a) Find the rank of the singular matrix of order 4×4 [2M]

b) What type of the solutions exists for $2x + 3y = 5,\ 4x + 6y = 10$ system? [2M]

c) If 5 is an Eigen value of A the find the Eigen value of $4A + 5I$ [2M]

d) Write the quadratic form associated with $\begin{bmatrix} 4 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ [2M]

e) Find the value of 'c' using Rolle's 's mean value theorem for $f(x) = x^2\ in\ [-1,1]$ [2M]

f) State Lagrange's mean value theorem. [2M]

g) Find $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ for $f(x, y) = log\sqrt{x^2 + y^2}$ [2M]

h) Find $\frac{\partial u}{\partial x}$ if $u = f(x + 2y, x - 2y)$ [2M]

i) If $f(x, y)$ be a continuous defined over a Region R, were [2M]
$R=\{(x, y)/\ x_1 \leq x \leq x_2$ and $c \leq y \leq d\}$ then $\iint_R f(x, y)\ dx\ dy = ?$

j) Evaluate $\int_0^2 \int_0^1 xy\ dx\ dy$ [2M]

PART – B (50 MARKS)

UNIT-I

2. a) Find the rank of the matrix using echelon form $\begin{bmatrix} 2 & 1 & -3 & -6 \\ 2 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$. [5M]

b) Solve the system of equations [5M]
$x + 2y + 3z = 0, 3x + 4y + 4z = 0,\ 7x + 10y + 12z = 0.$

(OR)

3 a) Test the consistency, if so, solve the system of equations [5M]
$5x + 3y + 7z = 4, 3x + 26y + 2z = 9, 7x + 2y + 10z = 5.$

b) Solve the system of equations using Gauss Seidel iteration method [5M]
$10x + y + z = 12, x + 10y - z = 10, x - 2y + 10z = 9$

UNIT-II

4. Reduce the quadratic form $2x^2 + 2y^2 + 2z^2 - 2yz - 2zx - 2xy$ to the canonical form byorthogonal reduction. Hence find nature, rank, index, and signature. [10M]

(OR)
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5. a) Find the Eigen values A^2 if $A = \begin{bmatrix} -1 & 0 & 2 \\ -1 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$ [5M]

b) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ [5M]

UNIT-III

6. Show that for any $0 < x < 1$,$x < sin^{-1}x < \frac{x}{\sqrt{1-x^2}}$ [10M]

(OR)

7. a) Verify Cauchy's mean value theorem $f(x) = sinx\ and\ \ g(x) = cosx$ in $\left[0, \frac{\pi}{2}\right]$ [5M]

b) Write the Taylor's series expansion for $f(x) = cosx$ about $x = \frac{\pi}{4}$ [5M]

UNIT-IV

8. a) Find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ if $u = \frac{x^2+y^2}{\sqrt{x}+\sqrt{y}}$ [5M]

b) Expand $f(x, y) = tan^{-1}\left(\frac{y}{x}\right)$ in powers of $(x - 1)\ and\ (y - 1)$ using Taylor's series. [5M]

(OR)

9. a) Find extreme values of $f(x, y) = x^2 - xy + y^2 + 3x - 2y + 1$ [5M]

b) Prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$ if $u = z\, tan^{-1}\left(\frac{x}{y}\right)$ [5M]

UNIT-V

10. By change into polar co-ordinates Evacuate $\int_0^a \int_0^{\sqrt{a^2-x^2}} (x^2y + y^2)\, dx\, dy$ [10M]

(OR)

11. Evaluate $\iiint_R (x^2 + y^2 + z^2)\, dx\, dy\, dz$, where R is the Region bounded by $x = 0$, $y = 0$, $z = 0$ and the sphere $x^2 + y^2 + z^2 = 1$ in the first octant. [10M]

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PART –A (20 Marks)

1\. a) The rank of 2×2 matrix with all elements are 3. [2M]

b) Write the condition for the homogeneous system of equations possess trivial solutions. [2M]

c) Find the nature of the quadratic form $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ [2M]

d) Find the Eigen values of A^T If 1 and 2 are the Eigen values of A. [2M]

e) Find the value of 'c' using Lagrange's mean value theorem for $f(x) = 2x \; in \; [0,1]$ [2M]

f) Write the Maclaurin's series. [2M]

g) Find $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ for $f(x, y) = e^x cos2y$ [2M]

h) Find $\frac{\partial u}{\partial y}$ if $u = f(2x + y, x - 2y)$ [2M]

i) If the region 'R' is divided into two sub regions, R_1, R_2 then $\iint_R f(x, y) \; dx \; dy = ?$ [2M]

j) Evaluate $\int_0^1 \int_0^1 \; dx \; dy$ [2M]

PART – B (50 MARKS)

UNIT-I

2\. a) Solve the system of equations [5M]
$x + 3y - 2z = 0, \quad 2x - y + 4z = 0, \quad x - 11y + 14z = 0$

b) Solve the system of equations using Gauss Jacobi iteration method [5M]
$10x + y + z = 12, \; x + 10y - z = 10, \; x - 2y + 10z = 9$

(OR)

3 a) Find the rank of the matrix using Normal form $\begin{bmatrix} 3 & 2 & -1 & 5 \\ 5 & 1 & 4 & 2 \\ 1 & -4 & 11 & -19 \end{bmatrix}$ [5M]

b) Test the consistency, if so, solve the system of equations [5M]
$x + y + z + t = 4, x - z + 2t = 2, y + z - 3t = -1, x + 2y - z + t = 3.$
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UNIT-II

4. Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ to the canonical form by orthogonal reduction. Hence find nature, rank, index, and signature. [10M]

(OR)

5. a) Find the Eigen values A^3if $A = \begin{bmatrix} 3 & -2 & 2 \\ 6 & -4 & 6 \\ 2 & -1 & 3 \end{bmatrix}$ [5M]

b) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix}$ [5M]

UNIT-III

6. Show that for any $x > 0$,$1 + x < e^x < 1 + xe^x$ [10M]

(OR)

7. a) Verify Cauchy's mean value theorem $f(x) = x^2$ *and* $g(x) = x^3$ in [1,2] [5M]

b) Write the Taylor's series expansion for $f(x) = sinx$ about $x = \frac{\pi}{4}$ [5M]

UNIT-IV

8. a) Prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$ if $u = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$ [5M]

b) Find the maximum and minimum distance of the point $(3,4,12)$ from the sphere $x^2 + y^2 + z^2 = 1$ using Lagrange's multiplier method [5M]

(OR)

9. a) Find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ if $u = log\left(\frac{x^2+y^2}{xy}\right)$ [5M]

b) Find extreme values of the following functions$f(x, y) = xy(a - x - y)$ [5M]

UNIT-V

10. Evaluate $\iint_R(\sqrt{xy} - y^2)\ dx\ dy$ where R is a triangle with vertices (0,0), (1, 0), (1, 1) [10M]

(OR)

11. Find the volume under the parabolic $x^2 + y^2 + z = 16$ over rectangle $x = \pm a$, y = ±b [10M]

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